Solving Probability Puzzles with SWARM’s inbuilt Calculator

This article is a brief guide to solving probability puzzles in the simplest possible way, which is to use SWARM’s inbuilt probability calculator.

For an overview of probability puzzles and the SWARM approach, see the overview article, Introduction to Handling Probability Puzzles. Also, consult or review the SWARM Probability Calculator Tutorial as needed for basic formulas and such. (Remember the calculator is limited to True/False variables right now.)

The Procedure

Write the provided information as Calculator statements

Write the questions as a Calculator queries

Press “Calculate” to view the results

Do a “sanity check”

The procedure for solving probability puzzles using the calculator is really just writing the probability information and questions in the SWARM response editor in a format the Calculator can understand.

Then you click on the Calculate button on the edit toolbar to see the results.

Click on the Reset button to remove the results and be able to edit the probability formulas (statements and queries).

It is easy to make mistakes, so do a “sanity check” on the results. You’ll probably need to do some fine-tuning of the formulas you’ve written.
A simple example

Let’s see how this works with the simple example we’ve seen previously, the Grenco/Blue Dynamics puzzle.

You are watching and analyzing drone surveillance activity. The first thing this morning your screen shows that one of our drones was involved in an accident. The commander needs to know immediately which of our drones was involved. We have two types of drones on this battlefield: the Grenco Air and the Blue Dynamics. You know that 85% of our drones are Grenco and 15% are Blue. A forward observer emails you that she has identified the drone as Blue. You note that it was getting dark as the forward observer saw the accident. Your Big Database tells you that the reliability of the forward observer under the same circumstances that existed on the night of the accident identifies each one of the two drones 80% of the time and failed 20% of the time.

Q. What is the probability that the drone involved in the accident was one of our Blue Dynamics drone?

The influence diagram for this puzzle is very simple. We use names like “DroneBlue” instead of “Drone” because the calculator treats everything as True/False:

Step 1: Write the provided information as Calculator statements

We use percentages for clarity, so “15%” instead of “0.15”. Either is acceptable. We also use “chance”, though “probability”, “prob” and “pr” also work.

<table>
<thead>
<tr>
<th>Information provided</th>
<th>The same information expressed in Calculator</th>
<th>What will be displayed after pressing Calculate</th>
</tr>
</thead>
<tbody>
<tr>
<td>85% of our drones are Grenco and 15% are Blue Dynamics</td>
<td>‘chance of DroneBlue is 15%’</td>
<td>chance of DroneBlue is 15%</td>
</tr>
</tbody>
</table>
the reliability of the forward observer ...identifies each one of the two drones 80% of the time and failed 20% of the time  

| the forward observer has identified the drone as Blue | `chance of ReportBlue given DroneBlue is 80%`  
| chance of ReportBlue given not DroneBlue is 20%`  
| chance of ReportBlue given DroneBlue is 80%  
| chance of ReportBlue given not DroneBlue is 20%  

Nothing! This is evidence to be provided during a query. Do not do this: `chance of ReportBlue is 100%`  

Steps 2 & 3: Write the question(s) as a Calculator queries

<table>
<thead>
<tr>
<th>The question</th>
<th>The question expressed in Calculator (a “query”)</th>
<th>After pressing Calculate</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the probability that the drone involved in the accident was one of our Blue Dynamics drone?</td>
<td><code>percent chance of DroneBlue given ReportBlue</code></td>
<td>41%</td>
</tr>
</tbody>
</table>

Best practice: use the “?” and “%” shortcuts

Adding “?” to the end makes Calculator include the statement in the answer, so your team can see what you did. (But it may drop highlights.) Using “%” instead of “percent” makes Calculator include the “%” sign in the answer.

<table>
<thead>
<tr>
<th>The query</th>
<th>After pressing Calculate</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>chance of DroneBlue given ReportBlue</code></td>
<td>0.41379310344827586</td>
</tr>
<tr>
<td><code>percent chance of DroneBlue given ReportBlue</code></td>
<td>41%</td>
</tr>
<tr>
<td><code>percent chance of DroneBlue given ReportBlue?</code></td>
<td>percent chance of DroneBlue given ReportBlue? 41%</td>
</tr>
<tr>
<td>👍 <code>% chance of DroneBlue given ReportBlue?</code></td>
<td>% chance of DroneBlue given ReportBlue? 41%</td>
</tr>
</tbody>
</table>
Step 4: Sanity Check

Look at the results. Do they make sense? Correct answers might be surprising, so you need to think carefully.

1. **Are known answers correct?** Ask question with known answers, for example:
   a. `%chance of DroneBlue` should be 15.
   b. `%chance of ReportBlue given DroneBlue` should be 80.

   Always include a couple of “sanity check” questions with known answers. If these fail, see §Troubleshooting, below.

2. **Review the automatic diagram.** Select Diagram Tool followed by Calculator Diagram to see what network Calculator is using. Does it match your idea? (For example, is it [DroneBlue] → [ReportBlue] without extra nodes or reversed arrows?)

3. **How might the result still be correct?** In the drone problem, the surprisingly low value of 41% is because the chance of misidentifying a Grenco drone is high enough that we expect more misidentified Grenco drones than properly identified Blue Dynamics drones. Once you see this, it’s clear that (20% of 85) > (80% of 15).

4. **Did you make a mistake leading to a wrong result?** Re-check your formulas – both the probability statements, and the query, especially if you see False, 100, or 0 in the answers. See §Troubleshooting, below.

5. **Are the results stable?** Repeat the calculation a few times to ensure the results are within 1% point. (Mostly applies to older versions of Calculator that used sampling.; See §Troubleshooting.)

6. **Are they replicable?** Did another person, working independently, get the same result? Using a different method? There are more ways to get the wrong answer than to get the right answer, so independent replication is good evidence the answer is correct.

Finally, now that you know the answer, can you explain the core simply? For example, above we noticed that this problem comes down to (20% of 85) > (80% of 15). Taking it one step further, that becomes 17 > 12. The chance we want is 12 / (12+17) = 41%.

Or you can start with a large imaginary group, and successively cut away each step of the problem, following the Influence Diagram. In this case:

- 100 Drones (Imagine we have 100 total)
  - 85 are GrenCo, of which, 17 ReportBlue (20% false alarm rate)
  - 15 Blue Dynamics, of which, 12 ReportBlue (80% hit rate)

So, given ReportBlue, the odds are 12:17 for Blue Dynamics:GrenCo, or 41%.
We call this approach **Bayes trees** because we start with a single trunk or root, and at each step create another layer of branches, like this:

Typically we invert the tree to flow top-down not bottom-up, and we use cognitive tricks like only including the branches we need: in this case we know the witness said “Blue”. (See the help document on Bayes trees for more.)

**Troubleshooting**

**Simple and Conditional Variables**

The calculator can not (yet) handle variables that have *both* a simple chances (like `chance of X = 20%`) *and* dependencies (like `Chance of X given Y = 50%`). But you can get around this by conditioning your target on a *dummy* variable that holds the background, prior, or leak chance:

- If X has *both* causes *and* background rate, use a dummy variable:
  - Y → X: `pr X given Y = 50%`
  - Dummy: `pr Xdum = 20%` ← Dummy holds background chance
  - Dummy → X: `pr X given Xdum = 1` ← Link has probability 1

**Variable Dependency & Loops**

If you see **error: variable dependency issue**, then Calculator thinks you have defined a loop.

The most obvious is having both X→Y and Y←X like, `pr X given Y = 50%` and also `pr Y given X = 20%`. This can also happen if you try to give X both simple and conditional chances.
Weird Values

Be suspicious of 0 answers, especially if you are using raw probabilities (no “%” or “percent”). *Sometimes* a chance will be 0 (or near enough), but more often this will indicate a fault. For example, suppose we have defined:

- “chance of Rain is 20%”

Then see what happens with these queries (you can try them in the [Wetgrass example](#)):

<table>
<thead>
<tr>
<th>Query</th>
<th>Result</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>chance Rain?</code></td>
<td>chance Rain? 0.2</td>
<td>Works. 0.2.</td>
</tr>
<tr>
<td><code>chance Rain?</code></td>
<td>chance Rain? unknown variable in formula: Rain</td>
<td>Typo: like it says, &quot;Rain&quot; is an unknown variable.</td>
</tr>
<tr>
<td><code>chance Rain given Sun?</code></td>
<td>chance Rain given Sun? unknown variable in formula: Sun</td>
<td>&quot;Sun&quot; is undefined. Calculator warns you about this.</td>
</tr>
<tr>
<td><code>chance Rain given -Sun?</code></td>
<td>chance Rain given -Sun? unknown variable in formula: Sun</td>
<td>Same</td>
</tr>
<tr>
<td><code>chance Rain &amp; Sun = 10%</code></td>
<td>chance Rain &amp; Sun = 10% no variables given in the probability network Userspace</td>
<td>Can't use logical operators on left side of an &quot;=&quot; sign.</td>
</tr>
<tr>
<td><code>chance Rain &amp; Sun?</code></td>
<td>chance Rain &amp; Sun? unknown variable in formula: Sun</td>
<td>Previous statement failed and did not define &quot;Sun&quot;</td>
</tr>
<tr>
<td><code>chance of -Moon : Rain = 10%</code></td>
<td>chance of -Moon : Rain = 10% no variables given in the probability network Userspace</td>
<td>Logical operator on left of = again. ** &quot;Moon&quot; remains unknown.</td>
</tr>
<tr>
<td><code>chance and Sun or Rain?</code></td>
<td>chance and Sun or Rain? false</td>
<td>False because invalid statement. The &quot;and&quot; is misplaced.</td>
</tr>
</tbody>
</table>

Fails to Reset

*Calculation renders to preformatted text, but then gets stuck there, or consumes some of your formula during reset.*
Always use the **Reset** button before editing.

If Reset fails to solve the problem, **please report it**. There appear to be some edge cases where the editor entirely replaces the formula with the result. We need help tracking those down. (History: the “auto-switch” behavior is left over from previous versions where the editor lacked explicit Calculate & Reset buttons. It is a slightly different process.)

**Developers**: you may see this when writing in Markdown. The solution is to remove indents to avoid triggering Markdown's preformat mode.

**General Troubleshooting**

In general, if lines are giving odd answers or simply not parsing, most likely you have a typo, or you have asked Calculator something it cannot do. (At least ⅘ times in my testing.)

But it's possible you have hit an esoteric bug. Consider browsing the Github Issues Page to see if anything matches. Feel free to create an issue if you are so inclined, but primarily use regular SWARM support channels. That will alert more people and should be faster. It may also help your team (and SWARM) in scoring.

Any tool can let you down, and the more a tool does for you, the more likely it can lead you astray. That's one reason SWARM insists on using at least two methods. Calculator is a great way to find answers. Once found, can you explain them with Bayes trees? Can you solve it a different way in Calculator? Or use a Bayes net?

One trick: order of evidence should not matter. If you have two witnesses, the following are equivalent:

- Evaluate both witnesses together.
- Evaluate witness 1 and use the result as the starting probability (“prior”) for witness 2. Then evaluate witness 2.

In this way you can turn a complicated problem into a series of simpler ones.

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¹ My favorite was that “Crysalis” and “Alkemist” failed because the tokenizer recognized “is” as a keyword, even if it was inside a variable name! That took some work to find.
Stability

Older versions of Calculator used sampling. Starting with v3 the results should be stable from run to run, and fast on all CREATE-sized problems.

If you see results vary from run to run, let us know. It has been stable in our testing, and we want it to be robust. (We have seen a few cases where the answer is off by 1-2 percentage points -- like 32% instead of 30% -- but nothing in CREATE should depend on a 2%-point difference.

Won’t Calculate

*Calculator just sits there, or just spins*

Uncommon following the v3 update, but if it happens: (1) check for unbalanced backticks -- Calculator will not attempt to parse the document if there is an odd number of backticks; (2) check for incomplete or illegal statements like a bare `chance`; (3) Try a different browser. Calculator is developed on Chrome. We have seen an issue on Safari, though that should be fixed now.; (4) Notify the SWARM team.

Bad Model / Wrong Tool

Calculator makes some assumptions to make it easy to model typical problems. But there is no free lunch, so some probability puzzles are a bad fit for Calculator. For example, if we let P = Poison, A = Antidote, and D = Death, the following puzzle doesn't work right. Declare the probabilities:

```
`pr P = 50%`; `pr A = 25%`  (Each is likely.)
`pr D : P = 60%`            (P is 60% lethal. Alkaline.)
`pr D : A = 90%`            (A by itself is 90% lethal. Acid.)
`pr D given (P & A) = 30%`  (Together they are safer.)
```

Now calculate probability of D given all combinations of P & A, and compare to expected answers. (I am pasting the results after Calculate, with expected in [blue]. You would replace each cell with just the query, like `% pr D:-A&-P?`.)

<table>
<thead>
<tr>
<th></th>
<th>A = False</th>
<th>A = True</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = False</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = True</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The last one is way wrong compared to what we meant. But it makes sense on the independence model. Imagine we had 100 P&A events. Of these:

- There are 90 deaths from A alone. (90% lethal)
- Of the 10 remaining events, there are 6 deaths from P alone. (60% lethal)
- Of the 4 remaining events, there are 1.2 deaths from P&A together. (30% lethal)
- These add to 97 out of 100, or 97%

What’s going on?

Calculator excels for the typical case of independent causes where:

- The effect starts with a 0% chance of occurring.
- Each cause acts independently to cause the effect.
  - Typically you know the chance of the effect for each cause acting alone.
  - But you can know any combination of conditions. The rest are assumed 0%.
- All causal effects add.

Suppose there are exactly three ways (A, B, C) to die (D):

```
`pr A = 50%`
`pr B = 50%`
`pr C = 50%`
```

Calculator can model this very quickly, with only 6 probabilities:
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Calculation saves you 11 trivial probabilities that just make the alternatives sum to 100%, and 5 non-trivial probabilities saying how A, B, and C combine when several are present. It’s those last 5 that account for the additive behavior described here. Calculator assumes they have to add. That’s often, but not always, a good model.

If Calculator works, but some numbers seem way off, and none of the other fixes work, it may be that Calculator is a poor fit. Check especially whether there are reversing interactions, as seen here. If so you may have to create a full Bayes net, or some other method.

Or… you could try a clever hack.

The Negative Probability Hack

While I love this hack, consider that when you are spending your time hacking your tools to do things they’re not good at, you might just switch tools.

If D:(P&A) gets added to the individual values, then specifying a negative probability should get us what we need. The system will be at 96% and we want to be at 30%. So we need to find a probability \( p \) that takes us from 96% down to 30%, using only the remaining 4% (100 - 96 = 4). We have: \( .04 \ p = (30 - 96) \). Solving: \( p = -1650 \). So we can define the following.

| `pr P = 50%`; `pr A = 25%` (As before) | `pr D : P = 60%` (As before) | `pr D : A = 90%` (As before) | `pr D given (P & A) = -1650%` \( \leq \) Clever hack! |

Then we have the following (pasting after Calculate, with correct answer in [blue]):

<table>
<thead>
<tr>
<th></th>
<th>A= False</th>
<th>A = True</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = False</td>
<td>%pr D:-A&amp;P? 0% [0%]</td>
<td>%pr D:A&amp;P? 90% [90%]</td>
</tr>
<tr>
<td>P = True</td>
<td>%pr D:-A&amp;P? 60% [60%]</td>
<td>%pr D:A&amp;P? 30% [30%]</td>
</tr>
</tbody>
</table>
Next Steps

- Back to SWARM ZenDesk
- SWARM probability calculator tutorial